# MATH 1450 EXAM1

NAME Key — GRADE ———-OUT OF 15 PTS

Answer the following questions correctly (no decimal answer!) for a full credit.

1. **(1pt)** Find the ***points*** on the graph of $f(t) = t^3 - 12t + 4$ where the tangent line is horizontal.

(see notes) $f'(t) = 3t^2 - 12 = 0 \rightarrow t^2 = 4 \rightarrow t = \pm 2$

Hence the points $\boxed{(2, -12)}$ and $\boxed{(-2, 20)}$

2. **(1pt)** Find an **equation** of the tangent line to the curve $y = x^3 - x$, at $P(-1, 1)$.

(See notes & or activity)

when $x = -1$, $y = 1$

$y' = m(-1) = 3x^2 - 1 \Big|_{x=-1} = 3 - 1 = 2$ slope

$y = 2x + b \rightarrow 1 = 2(-1) + b \rightarrow b = 1 + 2 = b$

$b = 3$

So $\boxed{y = 2x + 3}$

3. **(1pt)** The position of a body at time $t$ seconds is $s(t) = t^3 - 6t^2 + 9t$. Find the body's **acceleration** each time the velocity is zero.

(see notes)

$s'(t) = 3t^2 - 12t + 9 = 0 \longrightarrow t^2 - 4t - 3 \Rightarrow \longrightarrow t = 3, 1$

$s''(t) = 6t - 12$

Now $\boxed{\begin{array}{l} s(1) = -6 \text{ m/s}^2 \\ s(3) = 6 \text{ m/s}^2 \end{array}}$

4. **(4pts)** Find the **derivative** of each function with respect to the variable for which it is defined: do **not** attempt to simplify your final answer.

i) $f(t) = \dfrac{1-t}{1+t}$    ii) $g(r) = (r^2 - r)^3$    iii) $k(x) = tan(x) + x^{1/4}$

iv) $q(x) = (x+2)(1-x)$    v) $g(z) = cos^{-1}(z-3)$    vi) $y = 2^x + \log_3(x)$

(i) $f'(t) = \dfrac{-1(1+t) - 1(1-t)}{(1+t)^2} = \dfrac{-2}{(1+t)^2}$ (Quotient rule)

(ii) $g'(r) = 3(-1+2r)(r^2-r)^3$ (Power rule)/(Chain rule)

(iii) $k'(x) = \sec^2 x + \frac{1}{4} x^{\frac{-3}{4}}$ or $\sec^2 x + \dfrac{1}{4\sqrt[4]{x^3}}$ (derivative of trig. function)

(iv) $q'(x) = (1-x) + (x+2)(-1)$

$= -2x - 1$

(v) $g'(z) = -\dfrac{1}{\sqrt{1-(z-3)^2}}$

(vi) $y' = 2^x \ln 2 + \dfrac{1}{x \ln 3}$

5. **(2.5pts)** Let $f(x) = x^2$ and $g(x) = \sqrt{x}$, find the following (leave your answers in a *simplified form*):

(notes) (a) $f(g(x))$ (b) $g(f(x))$ (c) $g'(x)$ (d) $\dfrac{1}{(f'(g(x)))}$ (e) $(f \circ g)'$.

(a) $f(g(x)) = [\sqrt{x}]^2 = x$ (b) $g(f(x)) = \sqrt{x^2} = |x|$

(c) $g'(x) = \dfrac{1}{2\sqrt{x}}$ or $\dfrac{x^{-1/2}}{2}$

(d) $\dfrac{1}{2\sqrt{x}}$

(e) $(f\circ g)' = [x]' = 1$

6. **(1pt)** If $h(x) = \sin x$, find in a *reduced fraction form*, $\left.\dfrac{d}{dx}h^{-1}\right|_{x=5/7}$

$(h^{-1}(x))' = \dfrac{1}{\sqrt{1-x^2}}$ and $(h^{-1})'$ at $x = 5/7$ is

$$\frac{1}{\sqrt{1-\left(\frac{5}{7}\right)^2}} = \frac{1}{\sqrt{1-\frac{25}{49}}} = \frac{7}{\sqrt{24}} \text{ or } \frac{7}{2\sqrt{6}}$$

7. **(1pt)** If $h(x) = \ln(x^2 - 3)$, find the exact value (*no decimal*) of the slope of the line normal to $h$ at the point $A(2, 0)$.

$h'(x) = \dfrac{2x}{x^2-3}$ at $x = 2$, we have

$m = h'(2) = \dfrac{4}{4-3} = \boxed{4}$, so the slope of the normal line is $\boxed{-\dfrac{1}{4}}$

8. (**3pts**) For each of the following equations, find $y'$. Leave each answer in a *simplified rational expression form.*

Notes
(a) $x^3 - 2xy + y^2 = 0$.

Hofs
(b) $y = sin(xy)$.

Study Guide
(c) $\ln x + \ln y = 7x - 7y$.

a

$$3x^2 - 2y - 2y'x + 2yy' = 0$$

$$y'(-2x + 2y) = -3x^2 + 2y$$

$$y' = \frac{2y - 3x^2}{2y - 2x} \quad \text{or} \quad \boxed{y' = \frac{3x^2 - 2y}{2x - 2y}}$$

b

$$y' = \cos(xy)(y + y'x)$$

$$y' - y'x\cos(xy) = y\cos(xy)$$

$$\boxed{y' = \frac{y\cos(xy)}{1 - x\cos(xy)}}$$

c

$$\frac{1}{x} + \frac{y'}{y} = 7 - 7y'$$

$$\frac{y'}{y} + 7y' = 7 - \frac{1}{x} \longrightarrow$$

$$y'\left(\frac{1}{y} + 7\right) = 7 - \frac{1}{x}$$

$$y' = \frac{7 - \frac{1}{x}}{7 + \frac{1}{y}} \quad \text{OR} \quad \boxed{y' = \frac{7xy - y}{7xy + x}}$$